Modeling Rainfall Runoff using 2D Shallow Water Equation

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Torrential storms often trigger flooding that causes damage in properties and loss of life. In this study a numerical simulation module is developed to enhance the capability of a 2D surface flow model, CCHE2D. Following the procedure for numerical model verification and validation of ASCE, the developed module is tested using both analytical solutions and experiment data.

The analytical solutions of kinematic wave equation for runoff occurring on a sloping plane subject to a constant rainfall of indefinite duration and finite duration were used to compare to the results of the numerical model with good agreements. Runoff processes measured in laboratory experiments were also simulated in this study using the 2D model. The simulated runoff processes and the observed physical processes again showed excellent agreements. These tests indicate that the CCHE2D model is capable of modeling rainfall-runoff and kinematic overland flows.

INTRODUCTION

Modeling rainfall-runoff is necessary to understand the physical process, predict what would happen on the ground and better protect the stormed areas from flooding and enhance public safety. When the rainfall intensity exceeds soil infiltration, water begins to accumulate on the ground surface and then flows as overland flow under the force of gravity. In order to simulate the rainfall-runoff process, the depth averaged shallow water equations known as Saint-Venant (SV) equations or 2D shallow water equations are usually applied. Zhang and Cundy (1989) used a finite-difference 2D shallow water model to simulate the rainfall-runoff experiments performed by Iwagaki (1955) in a three-slope laboratory flume. Shallow water models based on the depth averaged shallow water equations (2D-SWE) were extensively used to compute the flow field (Zhang & Cundy 1989, Kivva and Zheleznyak, 2005). 1D Kinematic wave theory has been used successfully to describe overland flows (Woolhiser and Ligget, 1967; Freeze, 1978; Cundy and Tento, 1985). Kinematic wave modeling requires the specification of geometry, kinematic equations, inflow, and initial and boundary conditions (Singh and Regl, 1981). Depending on the terms of the momentum equation which are considered, various approximations of these equations are used. The kinematic approximation is the simplest; where the friction slope is set equal to the bed slope and the pressure and inertial terms are ignored (Book et al, 1981).

In this study the model verification was carried out analytical solutions to compare the performances of the kinematic wave equations by Singh and Regl (1981) and Singh (1983). The first test case was derived using analytical solutions of kinematic equations for erosion occurring on a sloping plane which is subject to a constant rainfall of indefinite duration and the second test case was derived using constant rainfall of finite duration. Both test cases have been studied for a one dimensional plane.

In this paper, in particular two laboratory experiments used to compare the performances of enhanced numerical model. The first test case was obtained by Gottardi and Venutelli (2008) which involves a comparative analysis of 2D numerical models for overland flow simulations. The second test case obtained by Cea et al. (2008) presents some results which include rainfall runoff experimental results obtained in a 2D laboratory model.

Numerical solution scheme

A developed shallow water flow model called the CCHE2D (Jia et al. 2002) is used as the hydrodynamic flow model for simulating the rainfall-runoff overland flow. CCHE2D is a hydrodynamic model for unsteady turbulent open channel flow and sediment transport. The governing equations for hydrodynamics are as follows:

$$\frac{\partial h}{\partial t}+\frac{\partial uh}{\partial x}+\frac{\partial vh}{\partial y}=R \quad (1)$$

$$\frac{\partial uh}{\partial t}+\frac{\partial uuh}{\partial x}+\frac{\partial vuh}{\partial y}=-gh\frac{\partial \eta}{\partial x}+\left(\frac{\partial h\tau_{xx}}{\partial x}+\frac{\partial h\tau_{xy}}{\partial y}\right)-\frac{\tau_{bx}}{\rho}+f_{Cor}v \quad (2)$$

$$\frac{\partial vh}{\partial t}+\frac{\partial uvh}{\partial x}+\frac{\partial vvh}{\partial y}=-gh\frac{\partial \eta}{\partial x}+\left(\frac{\partial h\tau_{yx}}{\partial x}+\frac{\partial h\tau_{yy}}{\partial y}\right)-\frac{\tau_{by}}{\rho}-f_{Cor}u \quad (3)$$

where u,v depth-integrated velocity components in x and y directions, g the gravitational acceleration, η is the water surface elevation, h is the local water depth f_{Cor} is the Coriolis parameter, T_{xx}, T_{xy}, T_{yx}, T_{yy} are depth integrated Reynolds stresses, T_{bx}, T_{by} shear stresses on the bed, R rainfall intensity. The 2D shallow water equations are solved using mixing finite element and finite volume methods with structured rectangular grid. Partially staggered grid is used for solving these equations. When runoff process is computed, the turbulence stress terms are neglected, because under this condition, the dominant forcing of the flow is the gravity term, momentum advection and bed shear stress. In the present simulation the Manning formula has been used to express the bed friction as

$$\tau_{bx}=\frac{1}{h^{1/3}}\rho g n^2 uU \quad (4)$$

$$\tau_{by}=\frac{1}{h^{1/3}}\rho g n^2 vU \quad (5)$$

Because

$$\frac{\partial \eta}{\partial x}=\frac{\partial h}{\partial x}+\frac{\partial b}{\partial x} \qquad \frac{\partial \eta}{\partial y}=\frac{\partial h}{\partial y}+\frac{\partial b}{\partial y} \quad (6)$$

where, h is the local water depth and b is the thickness of the bed. When runoff is simulated, the water depth is very small and parallel to the runoff slope, one has

$$\frac{\partial \eta}{\partial x}\approx\frac{\partial b}{\partial x} \quad \text{and} \quad \frac{\partial \eta}{\partial y}\approx\frac{\partial b}{\partial y} \quad (7)$$

Equation (2) and (3) are simplified approximately to kinematic wave equations. Therefore they can be tested using analytical solutions for the kinematic wave equation. The general forms of these equations make them applicable for general flow conditions.

Analytical solution

The analytical solution for the model tests was obtained by Singh and Regl (1981) and Singh (1983), for solving one-dimensional kinematic equation for rainfall generated runoff. The first test case involves analytical solutions of kinematic wave equation for runoff occurring on a sloping plane subject to a constant rainfall of indefinite duration and the second test case uses the constant rainfall of finite duration. The governing one dimensional kinematic equation can be obtained be simplification of Eq. (1) and (2), and written as:

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}=R \quad (8)$$

$$u=\alpha h^{n-1}; \quad Q=uh=\alpha h^{n} \quad (9)$$

where h is depth of flow *(m)*, u velocity of flow *(m/s)*, Q discharge of water per unit width *(m²/s)*, R lateral inflow or the effective rainfall *(m/s)*, α depth-discharge coefficient m^{2-n}/s and η an exponent (=5/3) Substituting Eq. (9) into Eq. (8) , the kinematic-wave equation can be then written as:

$$\frac{\partial h}{\partial t} + n\alpha h^{n-1}\frac{\partial h}{\partial x} = R \tag{10}$$

Table 1 shows the conditions of the two analytic cases.

The analytical solution described above has been verified using numerical model. Figure 1 and Figure 2 show the comparisons of the analytical solutions. In the numerical simulation, verification is necessary because one must need to assure that the numerical model is free of faults in mathematical formulations. Figure 1 shows the runoff hydrographs of Case 1 at several locations of the slope including the downstream boundary, obtained by analytical solution by Singh and Regl (1981) and numerical solutions by CCHE 2D model. The mesh resolution affects the results slightly particular at the very downstream of the domain. Figure 2 showing the analytical solution (Singh, 1983) and simulated runoff hydrographs of Case 2 at several locations of the slope. Because this is a case with a rainfall of finite duration, the hydrographs have a difference pattern.

MODEL VALIDATION

The enhanced model is tested using four laboratory experiments. All of the cases are validated using the analytical solution and also using numerical solution of CCHE2D. The application carried out on impervious surface, so that the lateral inflow *R* coincides with the rainfall. Various situations are examined for the validation test, particularly the rainfall intensity variable in time is considered.

Test Case 1

This runoff laboratory experiments was conducted by Gottardi and Venutelli (2008). They proposed an accurate time integration method for the diffusion-wave and kinematic-wave approximated models for the overland flow obtained by using the second-order Lax-Wendroff and the three-point centered finite difference schemes. This simple example of flow was carried out along an inclined plane of length *L* = 200m and of unit width with uniform rainfall of *R* = 60 mm/h for *t* = 1 hr. The slope of the plane was 0.001 and Manning roughness n_m = 0.03 $m^{-1/3}$s . The time of concentration t_c, for this experiment, when the outflow equals the rainfall rate, is t_c = 31.6 min. Figure 3 shows the runoff hydrographs at the downstream boundary, obtained by the experimental case Gottardi and Venutelli (2008), analytical solution by Singh and Regl (1981) and by CCHE2D model. In Figure 3 the simulated processes and the observed physical processes showed excellent agreements and the arrival time and the maximum discharge are in good agreement with the analytical solution.

Test Case 2

Runoff laboratory experiments over simple geometries were also modeled recently by Cea et al. (2008). These experiments originally carried out by Iwagaki (1955) in a two dimensional geometry and used as a validation test in Cea et al. (2008). In this 2D rainfall-runoff test case, the watershed is a rectangular basin made of three stainless-steel planes (2m x 2.5m). Each of the planes has a slope of 0.05. Two dikes are located at a distance of 0.32m and 1.74m from the bottom left plane and 0.56m and 1.18m from top plane respectively. Height of the dikes was 1.86m and 1.01m respectively. Figure 4 shows the 3D mesh and the flow field near the dike. As the bed surface is impervious, infiltration was not involved for three test scenarios.

Test Case 2A

Three scenarios have been modeled using three different rainfall patterns. In the first scenario (test case 2A) rainfall intensity was 317 mm/h and the duration is 45s. Figure 5 shows the comparison between the numerical and experimental outlet hydrograph. The simulated processes and the observed physical processes showed excellent agreements. The shape of the hydrograph is well predicted and also the peak discharge.

Test Case 2B

In the second scenario (test case 2B) rainfall intensity was 320 mm/h, the rain has two peaks of 25s with 4 seconds apart. Figure 6 shows the comparison between the numerical and experimental runoff hydrograph. Again the simulated processes and the observed physical processes showed excellent agreements. The shape of the hydrograph is well predicted and also captured both of the peak discharge.

Test Case 2C

In the third scenario (test case 2C) rainfall intensity was 328 mm/h, similar to second test, but the rainfall paused for 7s before the second peak. Figure 7 shows the comparison between the numerical and experimental runoff hydrograph. The simulated processes and the observed physical processes showed excellent agreements. The shape of the hydrograph is well predicted and both of the peak discharges are captured well.

CONCLUSION

In this paper a comparative analysis a 2D shallow water model, CCHE2D have been performed to simulate rainfall runoff and overland free surface flows. The depth averaged mass and momentum conservation equations are solved, considering the effects of bed friction, bed slope and precipitation. For the verification and validation tests, analytical and experimental cases and numerical simulation results are presented. Spatial variation of rainfall is incorporated in the model and good agreement between the observation and simulation is obtained. The experimental validation of the model are also encouraging and indicated that the CCHE2D model is capable of modeling rainfall-runoff and kinematic overland flows. Future investigations will focus on more complex, real world scenarios such as watershed and urban flood simulation due to storm events as well as in the design of hydraulic structures to mitigate and control flood risks.

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Table 1. Rain rate and conditions for Figures 1 and 2

Test Case	Rainfall, *R* (m/s)	Depth discharge coefficient, *a* (m^{2-k}/s)	Manning, *n* ($m^{-1/3}s$)	Duration, T (s)
Case1 (Singh and Regl, 1981)	2.7 x 10^{-5}	5	0.02	1000
Case2 (Singh, 1983)	2.7 x 10^{-5}	5	0.02	200

Table 2. Rain rate and conditions for Figures 3, 5, 6 and 7

Test Case	Slope, S	Manning, *n* ($m^{-1/3}s$)	Rainfall, *R* (mm/hr)
Case1	0.001	0.03	60
Case 2A	0.05	0.02	317
Case 2B	0.05	0.02	320
Case 2C	0.05	0.02	328

Figure 1: Runoff hydrograph for analytical solution and numerical solution by CCHE 2D for rainfall of indefinite duration.

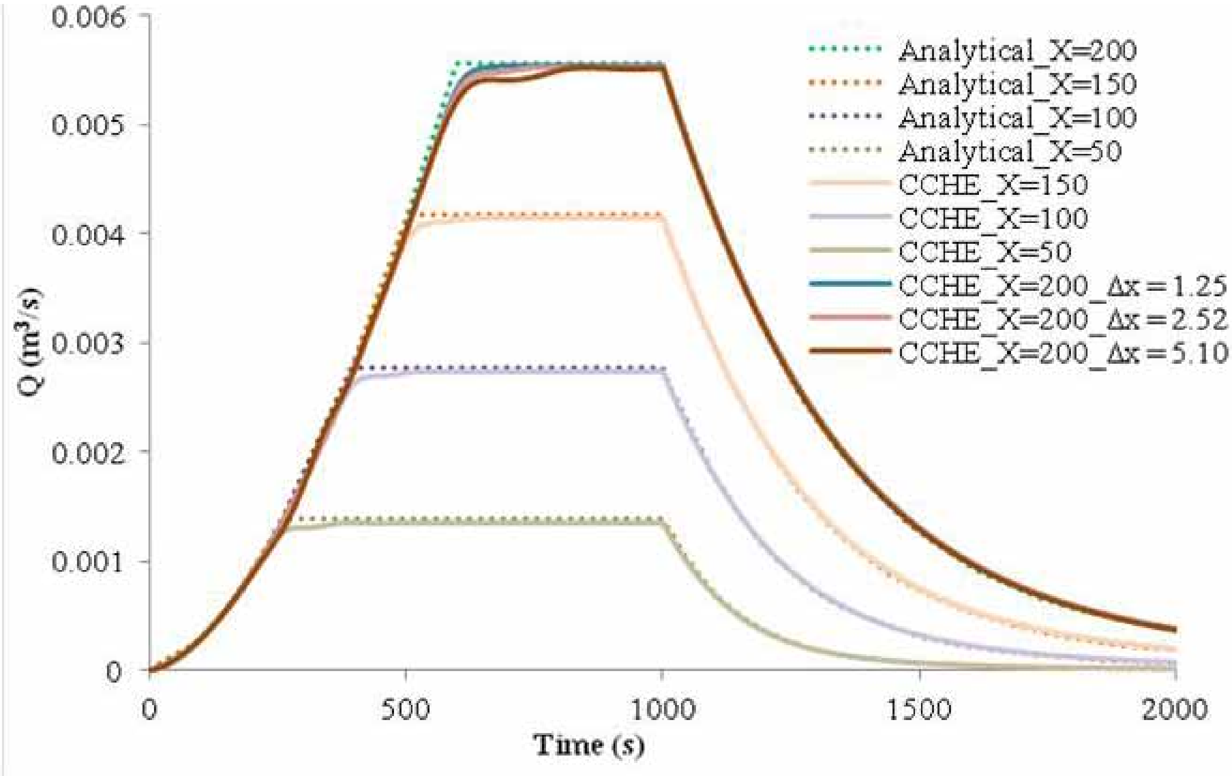

Figure 2: Runoff hydrograph for analytical solution and numerical solution by CCHE 2D for rainfall of finite duration.

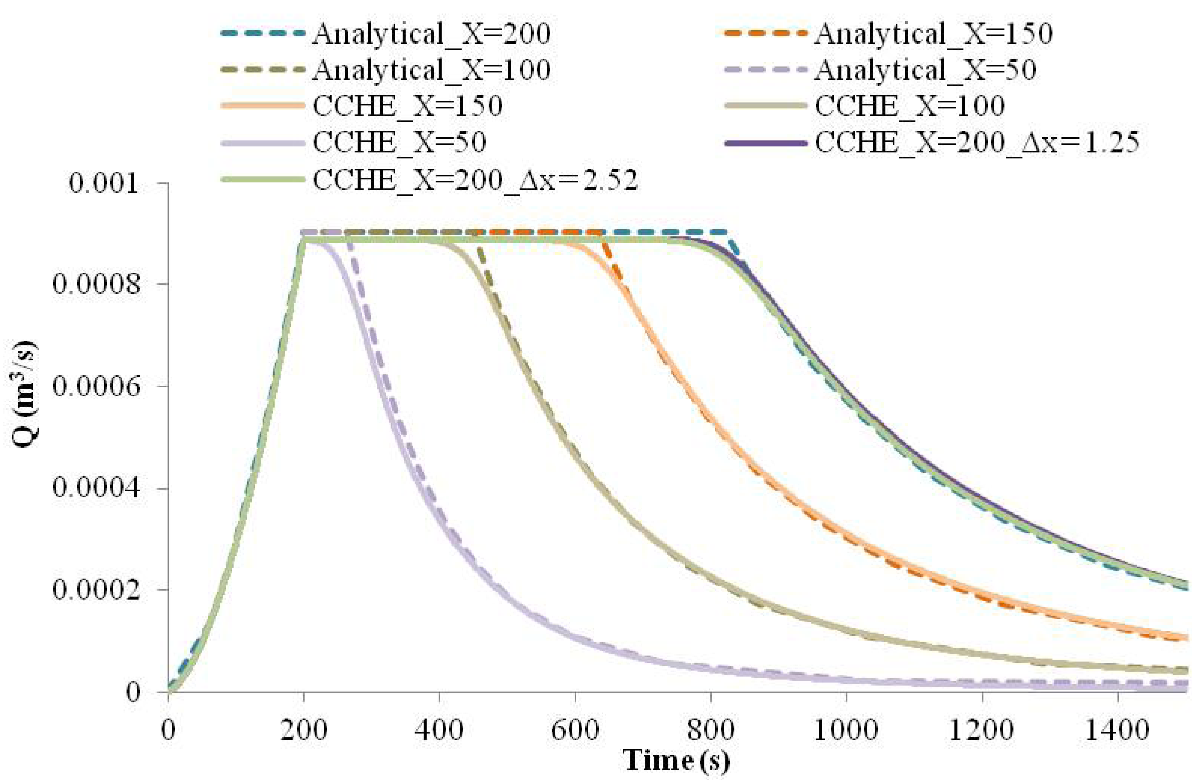

Figure 3: Runoff hydrograph for analytical solution, experimental data (Gottardi et al. 2008) and numerical solution by CCHE2D

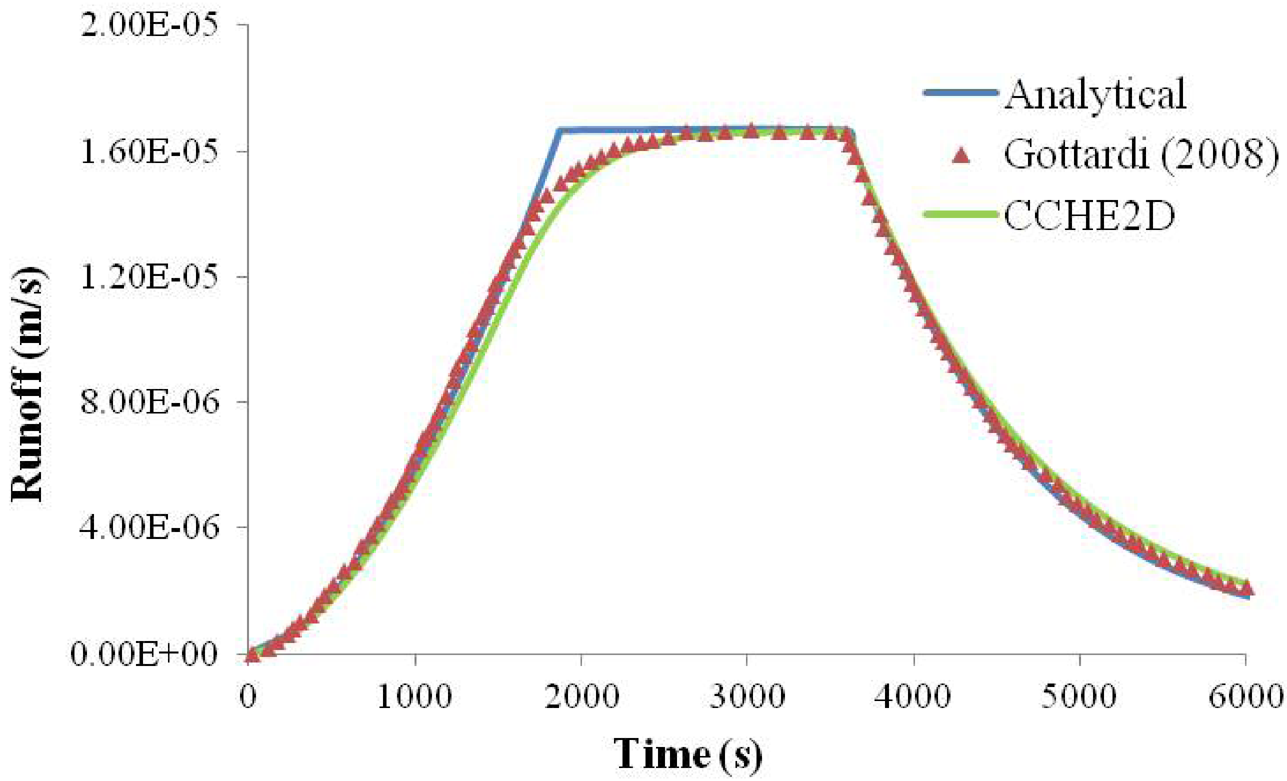

Figure 4: 3D mesh geometry (left) and water depth and velocity after the rain stops (T = 50s) (right) for test case 2

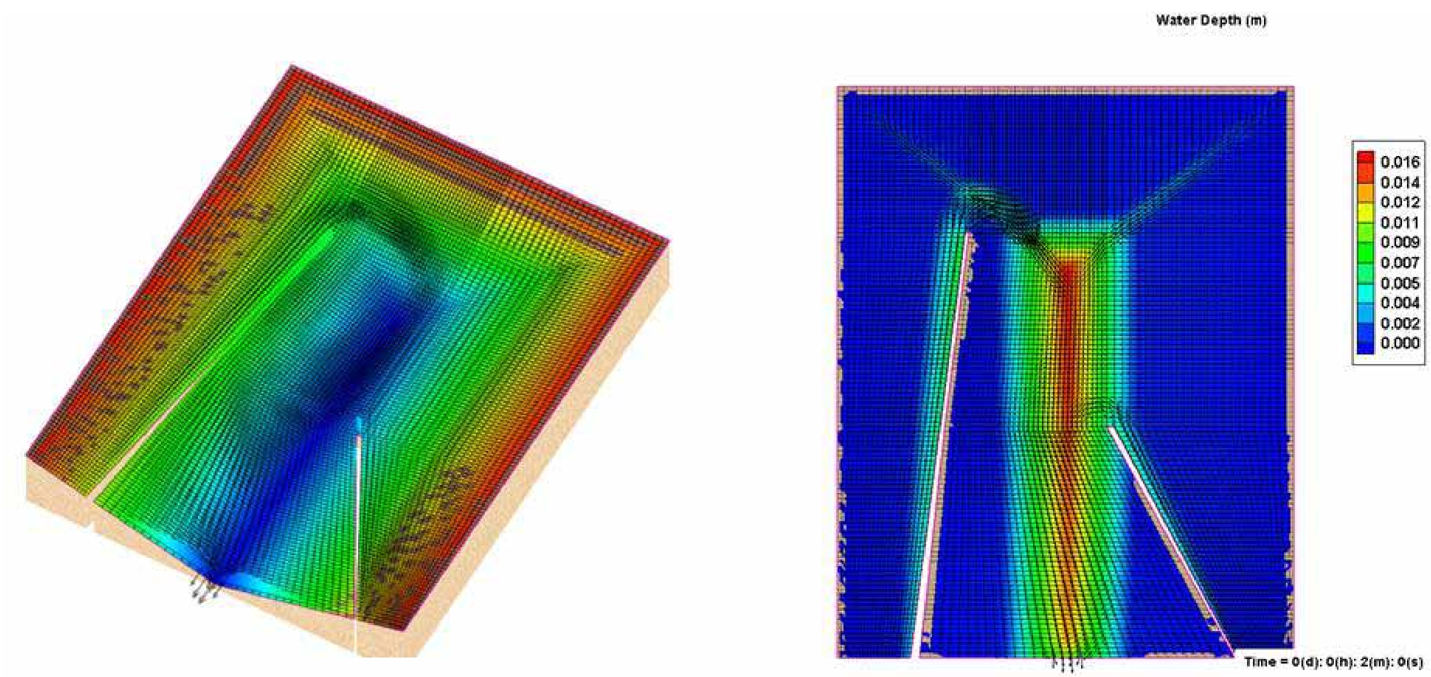

Figure 5: Runoff hydrograph for 2D validation test case 2A

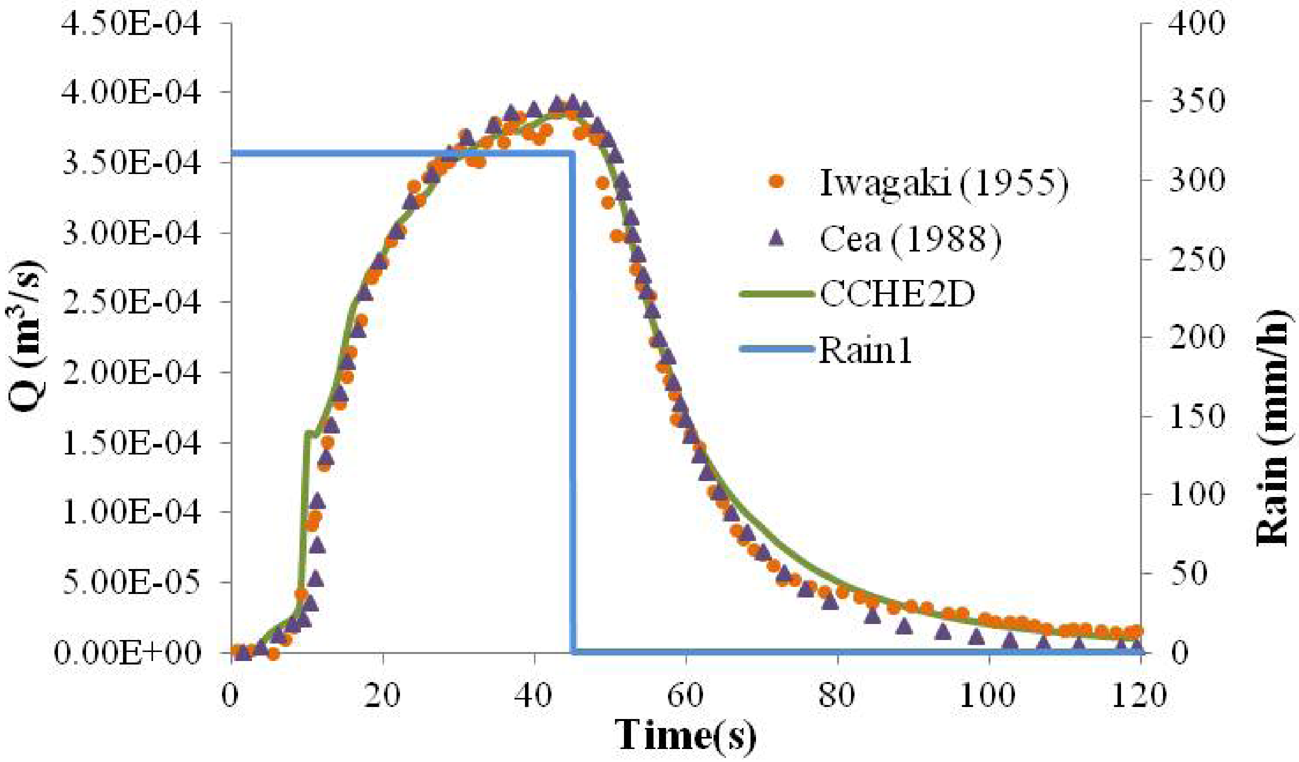

Figure 6: Runoff hydrograph for 2D validation test case 2B

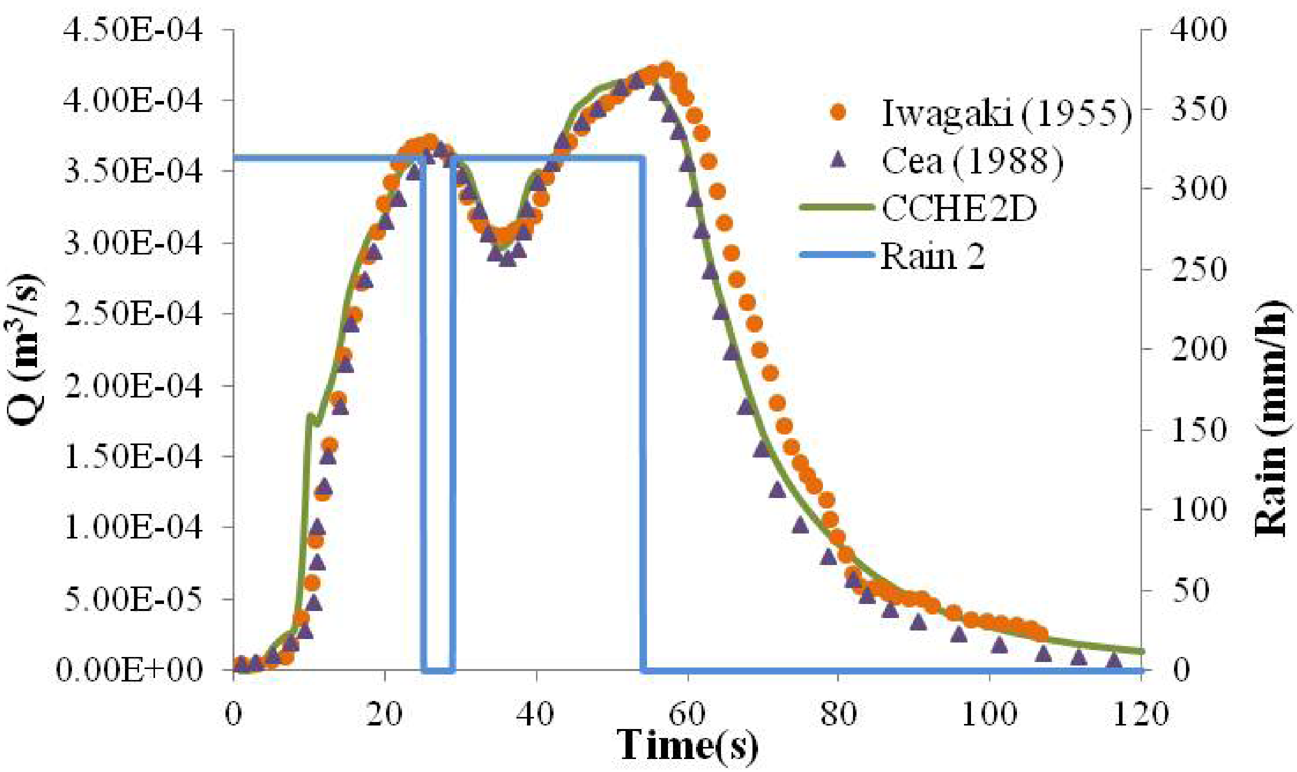

Figure 7: Runoff hydrograph for 2D validation test case 2C